

---

# **Modeling chemical reactions in immiscible fluids in microchannels**

**Olga Kuksenok and Anna C. Balazs**

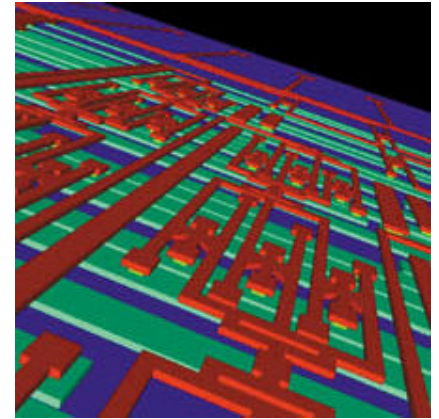
*University of Pittsburgh, Pittsburgh, PA*

# • Chemical Reactions in Microchannels: Motivations

---

## □ Microreactors

- ☞ Advantages: require nanoliters of substance, faster analysis
- ☞ Use for: drug screenings, advanced material preparation



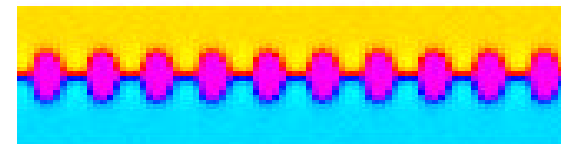
[www.ercim.org/.../enw43](http://www.ercim.org/.../enw43)

## □ Challenges

- ☞ Model reactions in immiscible fluids in microchannels
- ☞ Model spatially and temporarily non-uniform reactions
  - ☞ Photosensitive reactions

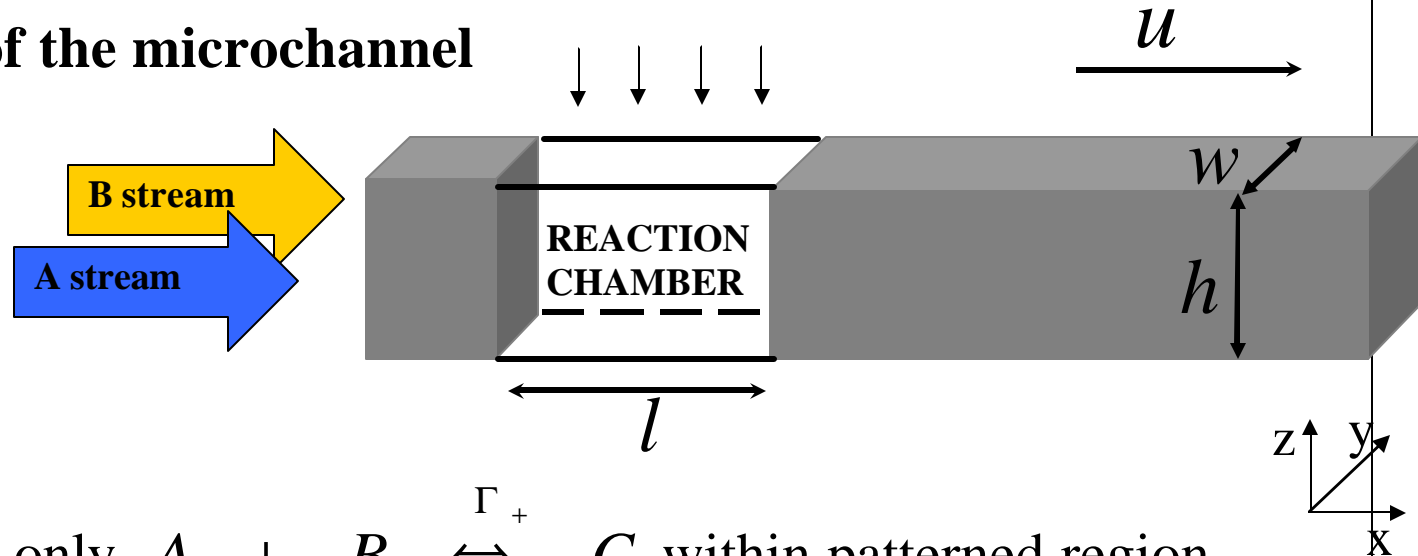
## □ Aim: control dynamics of immiscible fluids in microchannels using photosensitive reactions

- ☞ Create “necklace” of **C** droplets in **AB**



## • The Model

### □ Schematic of the microchannel



☞ Reaction only  $A + B \xrightleftharpoons[\Gamma_-]{\Gamma_+} C$  within patterned region

### □ Two order parameters

☞  $\phi = \rho_B - \rho_A$ ,  $\mathbf{y} = \mathbf{r}_c$ ;  $\mathbf{r}_i$  is concentration of  $i$ -component

☞ 
$$\sum_I \mathbf{r}_i = \mathbf{r}$$

☞ A, B and C are in three-phase coexisting region

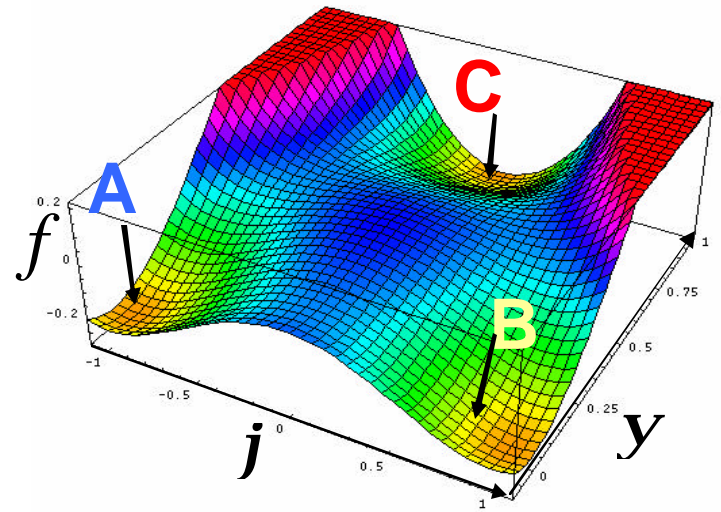
## • Ternary Mixtures

### □ Total free energy

$$F(\mathbf{j}, \mathbf{y}) = \int d\mathbf{r} \left[ f(\mathbf{j}, \mathbf{y}) + k_j (\nabla \mathbf{j})^2 + k_y (\nabla \mathbf{y})^2 \right]$$

☞ Local free energy density

$$f(\mathbf{j}, \mathbf{y}) = -A_{20}\mathbf{j}^2 + A_{40}\mathbf{j}^4 + A_{02}\mathbf{y}^2 - A_{03}\mathbf{y}^3 + A_{04}\mathbf{y}^4 + A_{22}\mathbf{j}^2\mathbf{y}^2$$



☞  $k_j, k_y$  define interfacial tensions,  $\mathbf{S}_{AB}, \mathbf{S}_{AC}, \mathbf{S}_{BC}$

☞  $\mathbf{S}_{AC} = \mathbf{S}_{BC} = \mathbf{S}_{AB}$

☞ If  $\mathbf{r}_c = 0$  reduces to binary phase-separating fluid

## • Evolution Equations

---

### □ Modified Cahn –Hilliard equations\*

$$\frac{\partial \mathbf{j}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{j} = M_j \nabla^2 \mathbf{m}_j$$

$$\frac{\partial \mathbf{y}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{y} = M_y \nabla^2 \mathbf{m}_y + \Gamma_+ (\mathbf{r} - \mathbf{y} + \mathbf{j})(\mathbf{r} - \mathbf{y} - \mathbf{j}) - \Gamma_- \mathbf{y}$$

☞  $M_j, M_y$  are mobilities

☞  $\mathbf{m}_j = dF / d\mathbf{j}, \quad \mathbf{m}_y = dF / d\mathbf{y}$  are chemical potentials

\* *C.Tong et al, J.Phys.Chem B, v106, 2002*

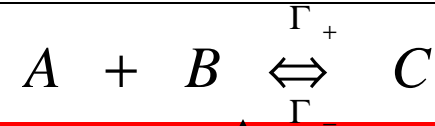
### □ Navier-Stokes equation: $0 = H + h \nabla^2 \mathbf{u} - \mathbf{y} \nabla \mathbf{m}_y - \mathbf{j} \nabla \mathbf{m}_j$

☞  $H$  is constant pressure gradient along  $x$

☞ Neglect  $\mathbf{y} \nabla \mathbf{m}_y, \mathbf{j} \nabla \mathbf{m}_j$  (high  $h$ )

### □ To solve use cell dynamic system method

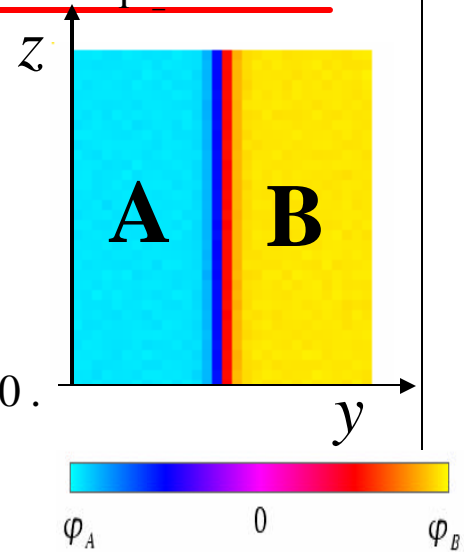
• **C Forms in “Reaction Chamber”**



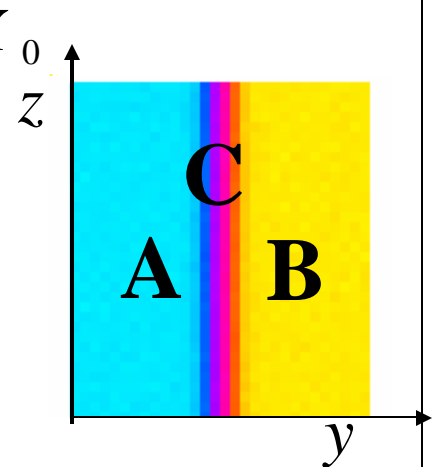
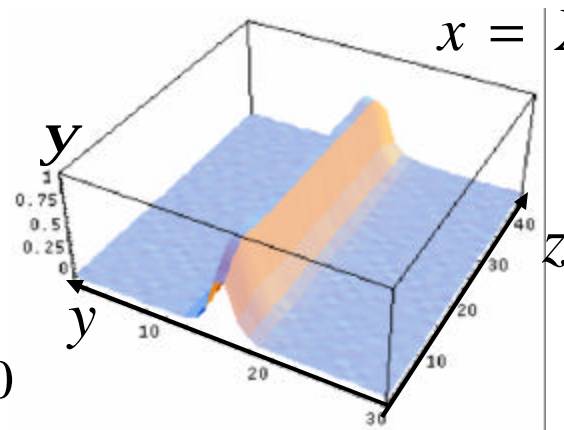
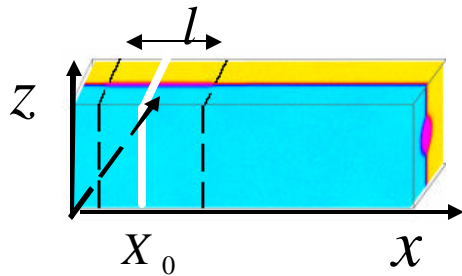
□ **Initial & boundary conditions:**

- ☞ At t=0: 2 streams of A & B , no C
- ☞ 2 streams A & B at inlet
- ☞ “Neutral” hard walls:

$$\left. \frac{\partial j}{\partial x} \right|_{x=L} = 0, \quad \left. \frac{\partial y}{\partial x} \right|_{x=L} = 0, \quad \left. \frac{\partial m}{\partial n} \right|_{wall} = 0, \quad \left. \frac{\partial m_y}{\partial n} \right|_{wall} = 0.$$



□ **C forms on AB interface in “reaction chamber”**



$$\Gamma_+ = 0.05, \quad \Gamma_- = 0.00125$$

box : 360 × 30 × 45 ; l = 50

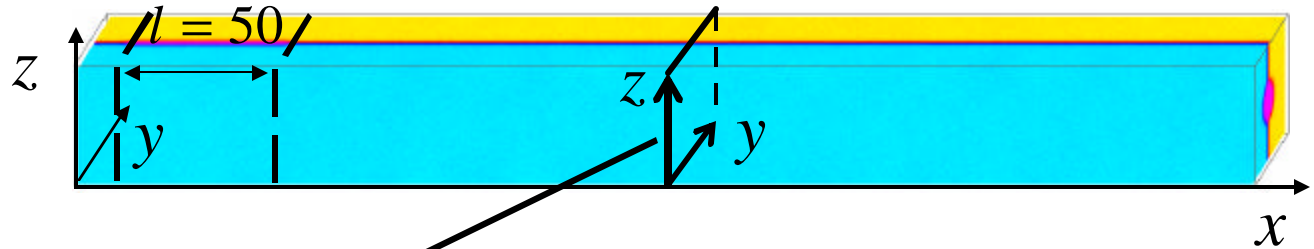
- ☞ Amount of C depends on  $\Gamma_+, \Gamma_-, s$        $\Gamma_+ \gg \Gamma_-$

• Evolution Within Microchannel ( $H = 4 \cdot 10^{-4}$ )

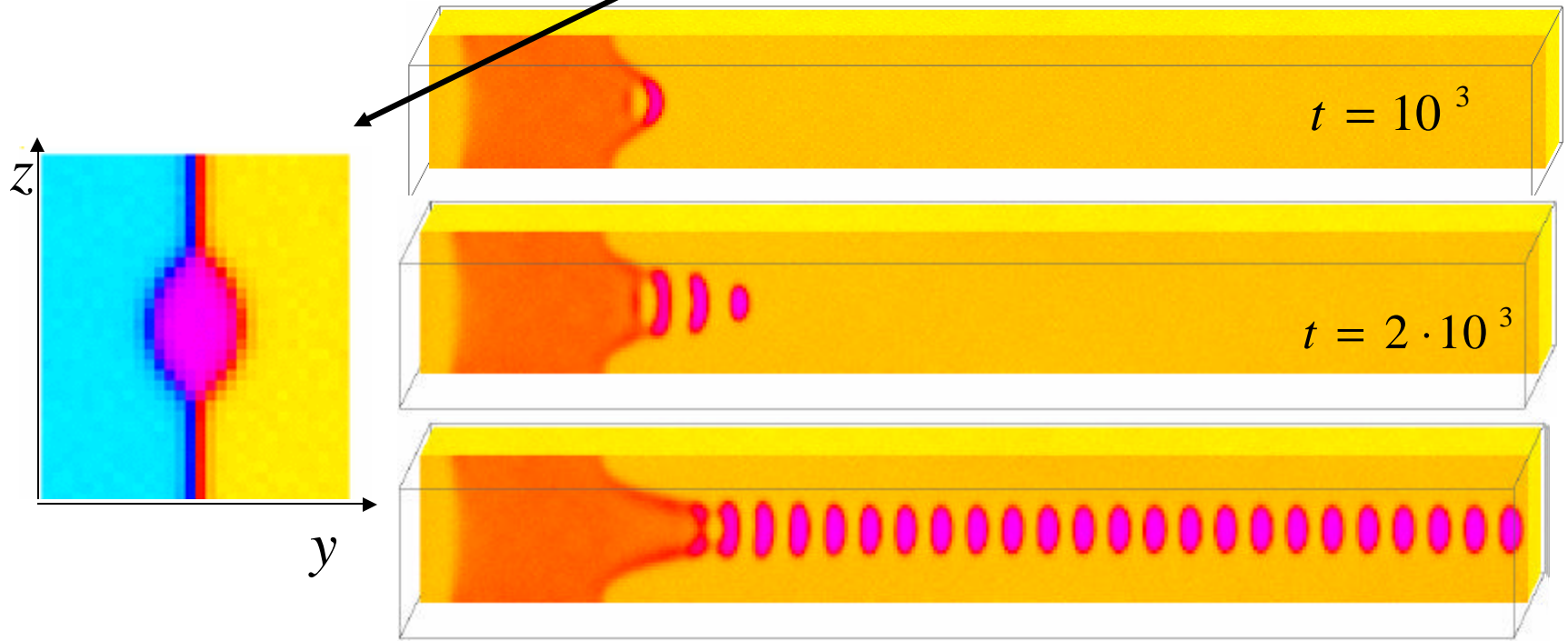
□ C layer is created on AB interface in “reaction chamber”

$$\Gamma_+ = 0.05$$

$$\Gamma_- = 0.00125$$



□ C is advected along x (below A is transparent,  $y = w/2$ )



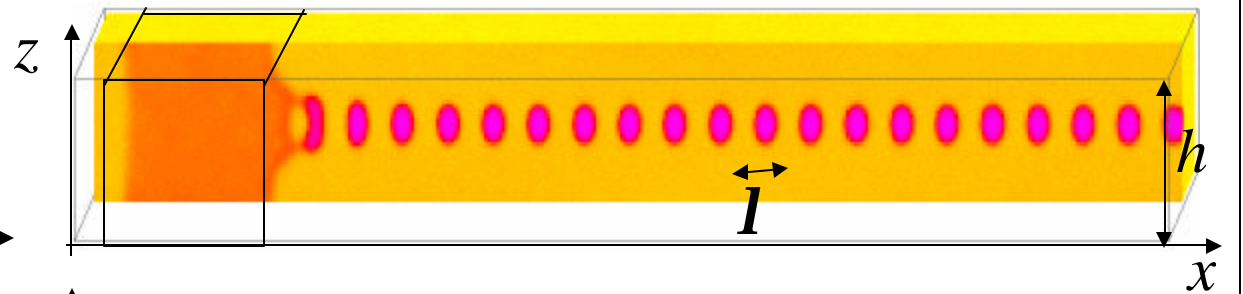
• **Dependence on Imposed Pressure Gradient,  $H$**

□  $H = 2.5 \cdot 10^{-4}$

$\Gamma_+ = 0.05, \Gamma_- = 0.0025$

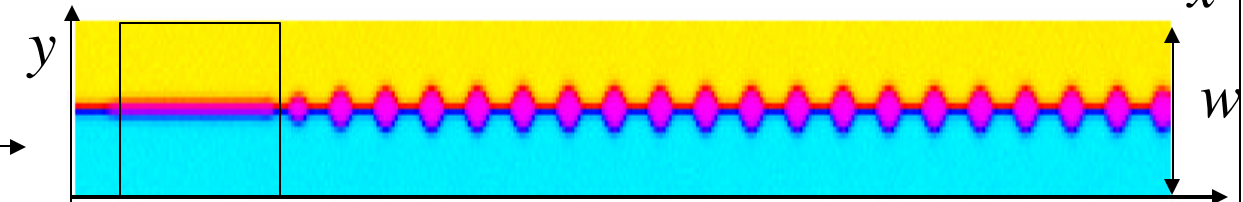
*Vertical X-section*

$(y = w/2)$



*Horizontal X-section*

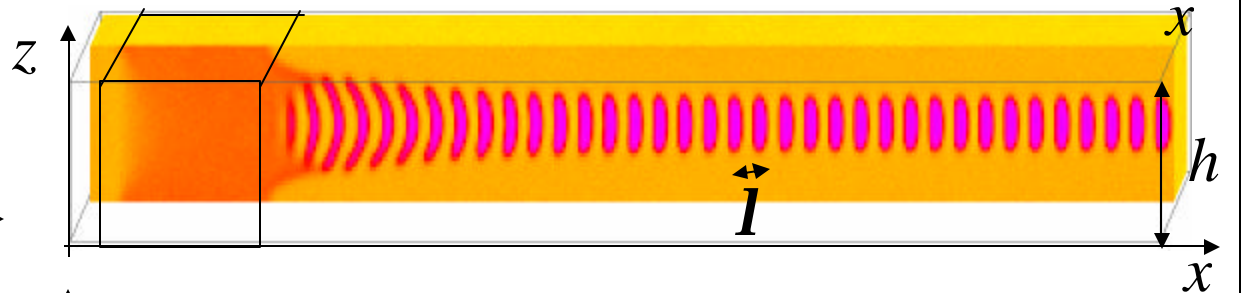
$(z = h/2)$



□  $H = 8 \cdot 10^{-4}$

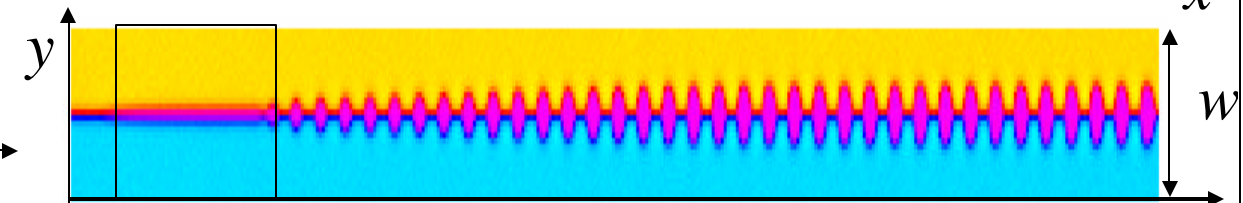
*Vertical X-section*

$(y = w/2)$



*Horizontal X-section*

$(z = h/2)$

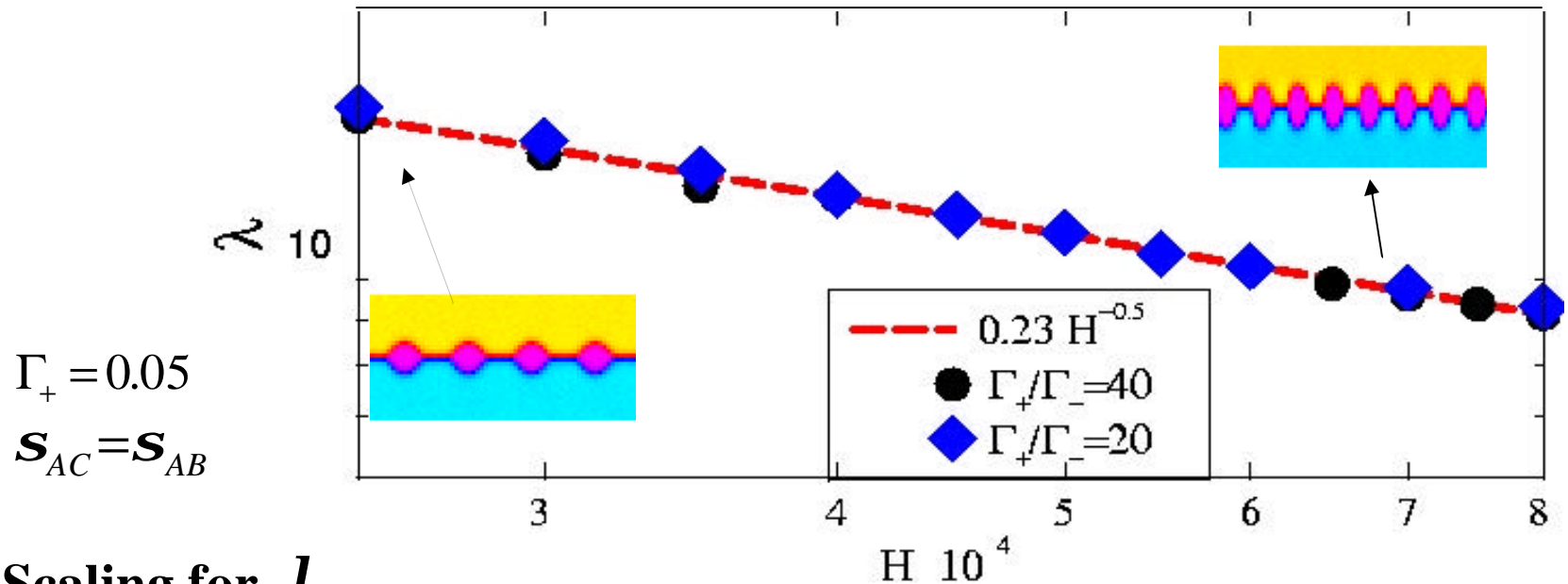


□  $l$  is smaller and droplets are elongated in  $y$  &  $z$  for higher  $H$



# • Dependence On Imposed Pressure Gradient, $H$

## □ Period in $x$ -direction, $l$



## □ Scaling for $l$

☞ Advection time  $t_{adv} \sim l / Hh^2$

☞ Time of C-droplet formation  $t_{diff} \sim M_y l^3 / \mathbf{s}_{AC}$

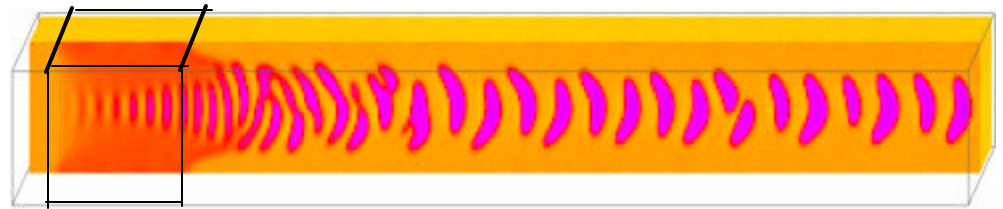
☞ Period in  $x$ :  $l \sim \frac{1}{h} \sqrt{\mathbf{s}_{AC} M_y / H}$

## • Chaotic Pattern Formation

---

Higher  $H$

$$H = 10^{-3}$$



☞ C-Layer within reaction chamber is strongly distorted

Lower interfacial tension

$$S_{AC} \ll S_{AB}$$



☞ C-Layer is advected longer (droplets do not form)

Higher  $\Gamma_+$  or  $\Gamma_- = 0$

☞ Too much C



Chaotic patterns are observed for wide range of parameters

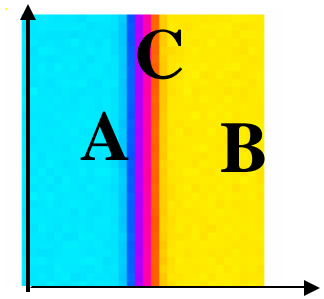
☞ To observe periodic patterns  $t_{diff} \approx t_{adv}$ ;  $l \ll w$ .

# • Time Dependent Reaction Rate Coefficient

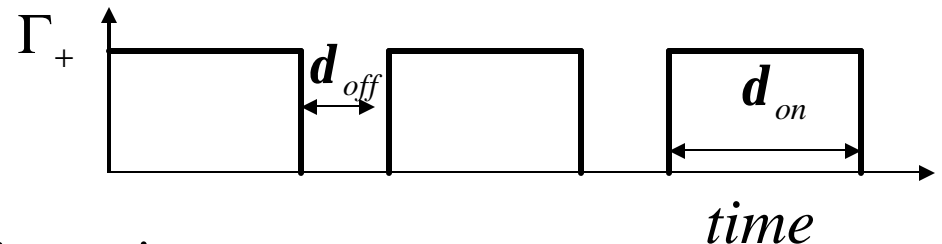
---

## ❑ Light is “continuous”

☞ Role of “reaction chamber”  
is equivalent to 3 streams at inlet



## ❑ Light is “blinking”



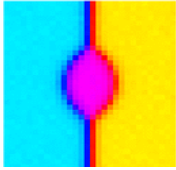
☞ *in-situ* control of pattern formation

$$H = 4 \cdot 10^{-4}$$

$$d_{on} = d_{off} = 10^3$$



☞ Sequences of droplets of different sizes

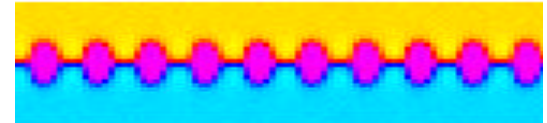


## • Conclusions and Future Plans

---

### □ Pattern formation in ternary systems within microchannels

- ☞ C droplets are formed periodically within A/B streams



### □ Future plans

- ☞ Time-dependent reaction (light on & off)



- ☞ *in-situ* control of pattern formation

- ☞ Surface reactions within “reaction chamber”  $\Gamma_+(z)$ ,  $\Gamma_-(z)$

- ☞ Effect of hydrodynamic interactions