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Director structure around a colloid particle suspended in a nematic liquid crystal

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We analyze the director field around a spherical colloid particle with radial (homeotropic) anchoring on its surface. Depending on the relative strength of anchoring W, controlled by the parameter WR/K with K the Frank constant and R the particle radius, the director distribution may possess a singular ring of a $-1/2$ disclination in the equatorial plane. The equilibrium radius of this ring, at rigid radial anchoring, is $a^* \approx 1.25R$ and only weakly depends on the disclination core energy. At small anchoring the director field is regular and weakly disturbed by the particle, there is a characteristic crossover W^* between the two regimes. We obtain the analytical expression for $\mathbf{n}(\mathbf{r})$, which decays as $r^{-3}\sin 2\theta$ away from the particle, and compare it with the exact numerical solution in the highly distorted nonlinear regime. [S1063-651X(96)04511-4]

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I. INTRODUCTION

Colloid systems attract a wide interest of academic and industrial researchers, in part due to a number of their important practical applications and also because the fluid dynamics processes, ordering and stability, of these systems present challenging and complex problems. The physics and chemistry of colloids and heterogeneous emulsions are rapidly becoming, from an empirical domain of paint and food industry, an exciting area of fundamental research [1,2]. Colloids are characteristically mesoscopic systems with structure and time scales such that typical shear rates can bring them out of equilibrium and into some exotic states. Of much interest are various interactions, occurring in colloids, for instance, and hydrodynamic and polymeric solvent-mediated forces.

Colloid suspensions in a liquid-crystal matrix have a crucial addition to their properties, which is the long-range deformation field created by particles in the liquid crystal due to the director anchoring on the particle surface. The analysis of this problem is difficult from both experimental and theoretical points of view. Experimentally, it is hard to visualize the director distribution in a thick sample of liquid crystal [3]. Perhaps the quenching and fracturing technique [4] is a more appropriate tool; otherwise one is forced to examine the secondary effects, such as the aggregation time or the anisotropy of the particle distribution. All such effects require a detailed knowledge of the director distribution around a colloid particle suspended in a liquid-crystal matrix. A full theoretical analysis of this problem is virtually impossible because of highly nonlinear problems in a complex geometry. A trial function $\mathbf{n}(\mathbf{r})$, based on the direct superposition of component deformations, has been exploited in Ref. [5]. It has been assumed that due to the rigid anchoring, the particle generates a director field of a radial hedgehog, to which one has to add a deformation of a $(-1/2)$ disclination ring to preserve the topological neutrality of the whole construction. As a result, a plausible director distribution (ansatz) can be constructed (see Fig. 1) with the director deviation angle given by

$$\beta_a(\mathbf{r}) = \arctan\frac{\rho}{z} - \frac{1}{2}\arctan\frac{\rho-a}{z} - \frac{1}{2}\arctan\frac{\rho+a}{z}$$
$$= \theta - \frac{1}{2}\arctan\frac{\sin 2\theta}{(a/r)^2+\cos 2\theta}, \tag{1}$$

where the first line is written in cylindrical coordinates $\{\rho,\phi,z\}$ and explicitly shows the component defect expressions (with a the radius of disclination ring). The second line is obtained by transformation to spherical coordinates $\{r,\theta,\phi\}$. Such an ansatz reflects the characteristic quadrupolar symmetry of the problem and the presence of a singular ring, but otherwise it is quite a poor approximation. It is easy to see that this trial distribution $\mathbf{n}(\mathbf{r})$ has its deformations decay $\delta\mathbf{n} \sim (r/a)^{-2}$ far away from the particle.

We shall argue below that the trial function (1) is qualitatively incorrect, and this article is devoted to obtaining a proper solution of the problem of a director distribution in a nematic liquid crystal around a spherical particle with radial (homeotropic) anchoring on the surface. We obtain this solution for different values of the nematic anchoring energy W and show that the characteristic $(-1/2)$ disclination ring disappears at a critical W^*, below which the director field is

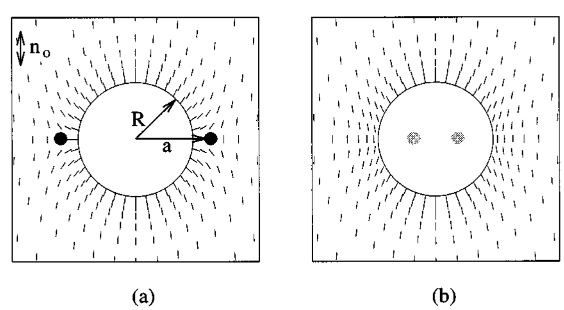

FIG. 1. (a) Particle (of radius R) with rigid radial boundary conditions and a disclination ring of radius a in the plane perpendicular to $\mathbf{n}_0$. (b) The particle with weak radial anchoring on the surface has a regular director distribution, which may be treated as an ''image'' disclination ring with $a \ll R$.

regular; in all cases the long-range asymptotic behavior of distortions is, in fact, $\delta\mathbf{n}\sim r^{-3}$ (Fig. 1).

The problem of director distribution and possible singularities in a heterogeneous liquid crystal has a more general relevance. The nematic liquid crystal, as any other vector-field system, is uniform in its ground state. However, in constrained geometries the structure with topological defects can be the ground state, and properties of such a state can be distinctly different from those of the uniform one. Constraints are most commonly imposed by the system boundary, which in this particular case is the particle surface (an ''inner boundary''). An immediate consequence of having inner boundaries in the system, including the possible disclination core surface, is that one has to be careful with surface terms in the Frank free-energy density. Normally, in the one-constant approximation one writes for it $\frac{1}{2}K(\nabla\mathbf{n})^2$, which is obtained from the general expression [6] by an integration by parts and dropping the surface contributions. Now we have to use the expression

$$F=\int\frac{1}{2}K[(\mathrm{div}\mathbf{n})^2+(\mathrm{curl}\mathbf{n})^2]d^3r, \tag{2}$$

which cannot be reduced any further. Strictly speaking, one also needs to add the two independent divergent elastic terms, which also have a contribution on the inner surfaces,

$$K_{13}\nabla(\mathbf{n}\ \mathrm{div}\mathbf{n})-K_{24}\boldsymbol{\nabla}(\mathbf{n}\ \mathrm{div}\mathbf{n}+[\mathbf{n}\times\mathrm{curl}\mathbf{n}]). \tag{3}$$

The effect of these terms has been extensively discussed in the recent literature [7]. This paper is aimed at obtaining the compact analytical solution for $\mathbf{n}(\mathbf{r})$ around the colloid particle. Therefore we will have to omit these terms, hoping that their effect will not change the conclusions in a qualitative way.

Finally, we define the anchoring energy W in the Rapini approximation [8]

$$F_s=-\oint\frac{1}{2}W(\mathbf{n}\cdot\hat{\boldsymbol{\nu}})^2dS, \tag{4}$$

with $\hat{\boldsymbol{\nu}}$ the unit vector normal to the particle surface. At rigid anchoring $W\to\infty$, the director will point radially at all points on this surface, thus forcing the system to respond to the topological mismatch by forming a disclination ring of opposite effective point charge. At moderate anchoring energy one should expect only small deviations of the director from its uniform undistorted orientation $\mathbf{n}_0$.

II. WEAK ANCHORING: A LINEARIZED SOLUTION

The task of finding a director distribution around our spherical particle consists of minimization of the Frank free-energy functional with boundary conditions, provided by it and the surface energy (4). Generally this class of problems is not solvable analytically because of its nonlinearity, brought in by the unit-vector constraint $|\mathbf{n}(\mathbf{r})|^2=1$. However, in the case of weak anchoring we expect only small deviations of the director and the problem can be linearized.

In the given geometry (Fig. 1) it is convenient to describe the director field by two principal angles of the spherical coordinate system $n_z=\cos\beta(\mathbf{r}), n_x=\sin\beta(\mathbf{r})\cos\phi$, and $n_y=\sin\beta(\mathbf{r})\sin\phi$, where ϕ is the azimuthal angle, thus respecting an obvious azimuthal symmetry of the problem. The differential equation for the Frank free-energy minimum in the one-constant approximation takes the form

$$\nabla^2\beta-\frac{\sin2\beta}{2r^2\sin^2\theta}=0, \tag{5}$$

with θ the polar angle of the spherical coordinate system, and the boundary condition on the particle surface $r=R$,

$$\frac{\partial\beta}{\partial r}=-\frac{W}{2K}\sin2(\theta-\beta). \tag{6}$$

Assuming that, at small anchoring, the director deviates from its uniform orientation $\mathbf{n}_0||\hat{z}$ by only a small amount $\beta(\mathbf{r})\ll1$, these equations transform into a linear problem:

$$\nabla^2\beta-\frac{\beta}{r^2\sin^2\theta}=0,\quad\left(\frac{\partial\beta}{\partial r}+\frac{\beta}{r}\right)_{r=R}=-\frac{W}{2K}\sin2\theta. \tag{7}$$

The general solution of Eq. (7), decaying at infinity, is

$$\beta=\sum_k\frac{C_k}{r^{k+1}}P_k^1(\cos\theta), \tag{8}$$

where P_k^1 is the associated Legendre polynomial. The boundary condition on the particle surface selects a particular mode $k=2$ with all other coefficients $C_{k\neq2}=0$. The director rotation angle takes the form

$$\beta=\frac{WR}{4K}\left(\frac{R}{r}\right)^3\sin2\theta. \tag{9}$$

Obviously, the approximation leading to this expression $\beta\ll1$ is satisfied when $WR/K\ll4$. The convention of ''weak anchoring'' usually corresponds to values $W\leqslant10^{-4}$ ergs/cm^2. Taking the typical value for Frank elastic constants $K\sim10^{-6}$ ergs/cm, this approximation is valid for particle sizes $R\ll0.5$ mm. Even for the conventionally ''strong anchoring'' $W\geqslant10^{-2}$ ergs/cm^2, colloid particles smaller than $R\sim5-6$ μm will satisfy the linearization approximation and the director field around them will be fully described by Eq. (9).

The decay rate of director deformations far away from the particle $\beta\sim r^{-3}$ is the consequence of the basic symmetry of this problem: the localized object with the origin of the spherical coordinate system in its center, floating in an otherwise uniform nematic, which provides the azimuthal (cylindrical) symmetry for $\mathbf{n}(\mathbf{r})$. There is a qualitative difference between the correct equation (9) and the trial function (1), which has the asymptotic behavior $\beta\approx\frac{1}{2}(a/r)^2\sin2\theta$ at $r\gg a$.

As a conclusion of this section, we maintain that in the regime of small distortions, which holds for weak anchoring on the particle surface, or at large distance from any particle where the memory of large director variations and singularities is lost, the director angle $\beta(\mathbf{r})$ is described by

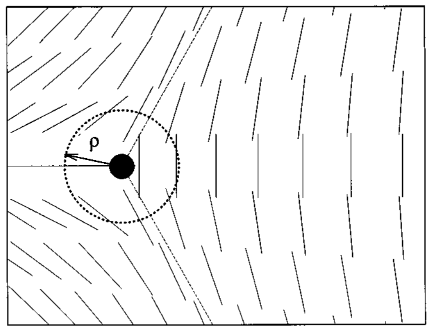

FIG. 2. Director distribution in the plane perpendicular to the disclination line, in the close vicinity of this line, $\rho/a \ll 1$. Dashed lines show the corresponding symmetry axes of a ''normal'' wedge $(-1/2)$ disclination.

the quadrupolar symmetry and cubic-power decay $\beta \rightarrow \mathrm{const}(R/r)^3 \sin 2\theta$. Let us note in passing that the same asymptotic behavior will be valid for the case of particles with planar anchoring, which corresponds to the W sign reversal in Eqs. (4), (6), and (9), but preserves the basic quadrupolar symmetry responsible for this type of behavior.

III. STRONG ANCHORING: A DISCLINATION RING

The opposite limiting case to the one considered in the preceding section is the regime of rigid radial anchoring on the particle surface. Now one cannot assume that $\beta \ll 1$ in the vicinity of the particle and there is no straightforward way to obtain the solution. An appealing possibility is to attempt to interpolate the far-field behavior (9): by comparison with Eq. (1) we can now write a new trial function

$$\beta_a = \theta - \frac{1}{2} \arctan \frac{\sin 2\theta}{(a/r)^3 + \cos 2\theta}. \tag{10}$$

Such a distribution gives the correct asymptotic behavior $\beta \sim r^{-3} \sin 2\theta$ as well as the ring of $(-1/2)$ singularity at $r = a$, $\theta = \pi/2$, which is a great advantage. However, the boundary condition on the particle surface $r = R$ is not respected by this $\beta_a(\mathbf{r})$.

Below we investigate the degree of accuracy of this trial function. First of all we obtain an exact numerical solution (in the one-constant approximation) for the director field, rigidly anchored with $\beta = \theta$ on the spherical particle surface and uniform $\beta = 0$ at infinity. The nucleus of the disclination ring is put in the boundary condition on the line $\theta = \pi/2$ in the form $\beta(r = a - \epsilon) = \pi/2; \beta(r = a + \epsilon) = 0$. The Appendix contains a brief description of the numerical relaxation method we used, which is analogous to the artificial compressibility method widely used in fluid dynamics (see, for instance [9]). As a next step, we try to find a unique function of the distance $f(r)$, which interpolates this exact numerical solution $\beta(\mathbf{r})$ in all relevant regions:

$$\beta(r,\theta) = \theta - \frac{1}{2} \arctan \frac{\sin 2\theta}{1/f(r) + \cos 2\theta}. \tag{11}$$

This assumption, that $f(r)$ is a unique function of r and independent of the angle θ, appears to be correct within reasonable limits of accuracy; see Fig. 4. Figure 5 shows the variation $f(r)$ for several values of the disclination ring size a; it is clear that the far-field limit $f(r) = (r/a)^3$ [Eq. (10)], adequately describes the director distribution at $r \ll a$. In the two other characteristic regions (the particle surface and the disclination ring) this function must comply with the conditions

$$r = R: \quad f(r) = 0 \quad \text{(i.e., rigid } \beta = \theta)$$

$$r = a: \quad f(r) = 1 \quad \text{(i.e., disclination at } \theta = \pi/2) \tag{12}$$

Obviously, the condition $f(a) = 1$ is satisfied even for the simple ansatz $f(r) = (r/a)^3$. However, the particle surface constraint produces the rapid variation of the real distribution between the ring and the surface.

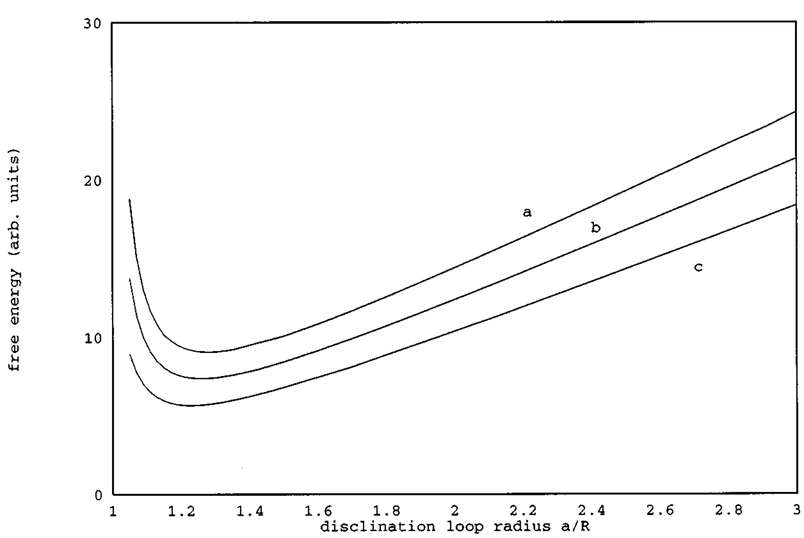

FIG. 3. Free energy of the system as a function of the disclination ring radius, scaled a/R. The three curves $F(a)$ correspond to the choice of the core cutoff (a) $r_c \sim 0.001R$; (b) $r_c \sim 0.01R$, and (c) $r_c \sim 0.1R$. It is quite clear that the optimal ring radius depends only weakly (if at all) on this choice: the minimum energy is achieved when $a^*/R \approx 1.25$.

It is possible to analyze the structure of the director field in the near vicinity of the disclination ring. Taking there $f(r)\approx 1+\alpha(r-a)$ and transforming to the local polar coordinates $\{\rho,\psi\}$ in the cross-section plane (Fig. 2) we examine the director texture around this singular line. One would expect, as the distance from the singularity decreases $\rho/a\to 0$, to recover a ''traditional'' straight $(-1/2)$ distribution $\beta=\text{const}-\frac{1}{2}\psi$. For this to happen another constraint on the universal function $f(r)$ must be imposed, namely, $[df(r)/dr]_{r=a}=2/a$. However, the comparison with the full numerical solution shows that this is not the case, the disclination in our construction is ''distorted'' with respect to the straight singular lines [6] (see Fig. 2). It is also interesting to compare this analysis with another known disclination ring: the Mori and Nakanishi ansatz for the circular $(+1/2)$ loop [10]. There the director field is assumed to follow the lines of the ellipsoidal coordinate system, which gives in the same local cross-section plane at $\rho/a\ll 1$ the exact straight wedge disclination expression $\beta_{m-n}=\text{const}+\frac{1}{2}\psi$. Therefore, we conclude that the presence of the radial particle in the middle and the total topological neutrality of the construction (and not just the fact that the disclination is encircled into a ring) make the director distribution near this singular line asymmetric.

By fitting the universal function $f(r)$ to the result provided by the numerical solution, we obtain an interpolated expression, which satisfies all the above constraints and allows us to work with the ''analytical'' form of $\beta(\mathbf{r})$, given by Eq. (11):

$$f(r)\approx\left(\frac{r}{a}\right)^3+\mathcal{A}+\mathcal{B}\,\frac{r}{a}+\mathcal{C}e^{-r/a}, \tag{13}$$

where the coefficients take the form

$$\mathcal{A}=\frac{R^3}{a^2(a-R)^2}\left[R-a+\frac{a^2}{R^2}(4a-3R)\left(\frac{a}{R}e^{-R/a}-e^{-1}\right)\right],$$

$$\mathcal{B}=\frac{R^3}{a^2(a-R)^2}[a-R+(4a-3R)(e^{-1}-e^{-R/a})],$$

$$\mathcal{C}=-\frac{4a-3R}{a-R}.$$

Clearly, this function becomes unstable at $a\to R$ since the solution is obtained in the assumption of rigid anchoring, which prevents placing the disclination near or on the surface. Although such an ultimate type of anchoring $(W\to\infty)$ is impossible in practice, it is still instructive to find the equilibrium ring radius under this assumption. Armed with the full numerical solution, as well as the interpolated analytical expression for $f(r)$, one can easily integrate the Frank free energy to obtain its dependence on the disclination size a. The result is plotted in Fig. 3. In this integration one faces the problem of the disclination core, which must be introduced in order to eliminate the relevant singularity (both for computational and physical reasons). The three curves $F(a)$ in Fig. 3 are obtained by choosing very different levels of the cutoff. The sharp increase of the free energy near the particle surface is determined by the rigid surface anchoring. The rise of the free energy at increasing a/R is due to the trivial increase of the disclination length $2\pi a$ since the energy of a loop is $\sim Ka(\ln[a/r_c]+\pi)$. Let us note that there is virtually no dependence on the disclination core radius r_c due to its appearance only under the logarithm. The disclination core (whether melted or biaxial, its energy density is always determined by the condition $E_c r_c^2=K$) is an independent physical parameter in this problem and cannot, as all other lengths, be scaled with the particle radius. Its effect can only be seen in the situation when the disclination is lying within the distance $\sim r_c$ from the surface; melting the nematic in this region will rectify the unphysical divergence in Fig. 3 at $a/R=1$, but does not change the value for the equilibrium ring radius $a^*\approx 1.25R$.

IV. DISCUSSION

We have examined the two limiting cases of the problem: the weak anchoring situation when the director deviations are small in the whole system and the exact analytical solution is possible and the rigid anchoring case characterized by the disclination ring. In both situations the far-field behavior of the director is identical and is described by the cubic-power decay of deformations. It is remarkable that even for the rigid anchoring case the regime when $f(r)=(r/a)^3$ with all possible accuracy extends practically to the region of the ring itself. For all practical purposes this approximation, expressed by Eq. (10), should be quite adequate.

An interesting question is to find out at what values of anchoring energy W does the crossover between the two above regimes take place. An explicit solution of this problem is very difficult because one would need to balance surface and bulk energies in the nonlinear regime, when the disclination is lying on the particle surface. Effects of the core size will have to be taken into account as well. Such a solution, however, is not necessary, especially after several approximations were made in Sec. III (no matter how good they are from our point of view), including the one-constant Frank elasticity limit and the neglect of surfacelike terms (3). Therefore we content ourselves with the qualitative estimate with this crossover value W^*. The simplest way to obtain it is by matching the two regimes of director variation

$$\beta=\frac{WR}{4K}\left(\frac{R}{r}\right)^3\sin 2\theta,\quad \beta=\left(\frac{a}{r}\right)^3\sin 2\theta. \tag{14}$$

This provides the estimate $W^*\sim 4Ka^3/R^4$. If we recall that the equilibrium ring size is $a\sim 1.25R$, this estimate becomes the expected $(W^*R/4K)\sim 2$, i.e., the border of applicability of the linearized solution.

It is possible, therefore, to observe the disclination ring as long as sufficiently big particles are used. One would suggest the use of a moderate-length main chain polymer liquid crystal, densely grafted on the particle surface to ensure strong radial anchoring. Then, by quenching the texture and fracturing or polishing the material off, one can reveal the region near the particle surface, where the disclination ring should be present in the equatorial plane, as Fig. 1 suggests. It is also important to calculate the total energy of deformations, created by the particle. This energy has been used, for instance, as a coupling term in the analysis of the phase equilibrium of nematic colloids [3]. Our model gives

$$F_c \approx \begin{cases} 0.2(W^2R^3/K), & \text{weak anchoring} \\ 6.7(Ka^6/R^5) \to 13KR, & \text{rigid anchoring.} \end{cases} \quad (15)$$

Comparing this result with the assumption made in [3], that the nematic-colloid coupling energy is $\sim KR \to \kappa Q^2$ with Q the nematic order parameter, one concludes that it corresponds to the case of rigid anchoring and is about an order of magnitude larger. However, at weaker anchoring (or for small colloid particles) the first of Eqs. (16) will be valid. This coupling energy has a very different dependence on Q: because the leading term of the anchoring energy is linear $W \sim Q$ (within the mean-field theory framework), the asymptotic behavior of F_c at $Q \ll 1$ is constant: $W^2/K \sim \text{const} + \alpha Q + \cdots$. Therefore, one may explore both the rigid and the effective weak anchoring regimes on decreasing the nematic order parameter Q, since the control parameter $WR/K \sim Q^{-1}$ and increases with $Q \to 0$. Changing the size of the particles is another simple tool to study both characteristic regimes of a liquid-crystalline colloid suspension.

ACKNOWLEDGMENTS

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APPENDIX: NUMERICAL SOLUTION AND INTERPOLATION FUNCTION

Numerical solution for the director distribution around the spherical particle with rigid radial anchoring on its surface reduces, after taking into account the symmetry of the problem, to the differential equation for polar angle $\beta(\mathbf{r})$ [Eq. (5)],

$$\nabla^2\beta - \frac{\sin 2\beta}{2r^2\sin^2\theta} = 0, \quad \text{(A1)}$$

in spherical coordinates $\{r, \theta, \phi\}$ with the origin in the particle center. For this type of problem it is favorable to use the spherical coordinates with an inverse scaled radius $\xi = R/r$, so that the grid in $\{\xi, \theta, \phi\}$ is more condensed near the particle surface, where one expects most of the variation to take place. At the same time one is capable of treating the infinite radius, corresponding here to the point $\xi = 0$. It is sufficient to consider the problem only in one quadrant of the cross section by the azimuthal plane, for $0 < \xi < 1$ and $0 < \beta < \pi/2$. The boundary conditions are imposed at $\xi = 1$, $\beta = \theta$; at infinity $\xi = 0$, $\beta = 0$; on the $\hat{z}$ axis $\theta = 0$: $\beta = 0$; and on the equatorial plane $\theta = \pi/2$, $\beta = \theta$ for $\xi < R/a$ and $\beta = 0$ for $\xi > R/a$ (the last constraint fixes the position of the singularity at $r = a$, between the two adjacent grid points).

The relaxation method is analogous to the artificial compressibility method widely used in fluid dynamics. The method uses the concept of an artificial temporal relaxation of the system with a chosen damping coefficient c:

$$\nabla^2\beta - \frac{\sin 2\beta}{2r^2\sin^2\theta} = -c^2\frac{\partial\beta}{\partial t}. \quad \text{(A2)}$$

The starting point (at $t = 0$) can be any director distribution, but it is advantageous to choose a plausible one, such as the ansatz (1), to speed up the computation. The distribution on every next time step of iteration is calculated according to Eq. (A2). Due to the damping introduced in this equation, the distribution will relax to a final steady state with no time dependence. This state, by definition, is the solution of the original equation (A1). In order to make this relaxation faster one should choose the coefficient c as large as possible, without causing instabilities of the numerical process. In this particular case, when c is the only independent parameter in

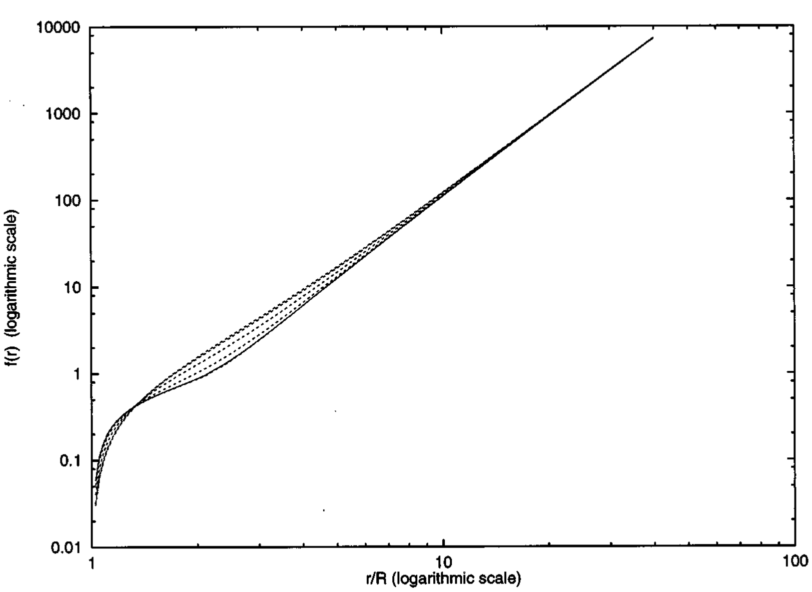

FIG. 4. Set of graphs for $f(r)$ for $a/R = 2.2$ and different values of the polar angle θ, which varies from $\theta = \pi/2$ (i.e., crossing the disclination line), solid line, to $\theta \to 0$ (i.e., along the $\hat{z}$ axis). This set indicates the degree of universality of the function $f(r)$ in Eq. (11), which apparently is very good.

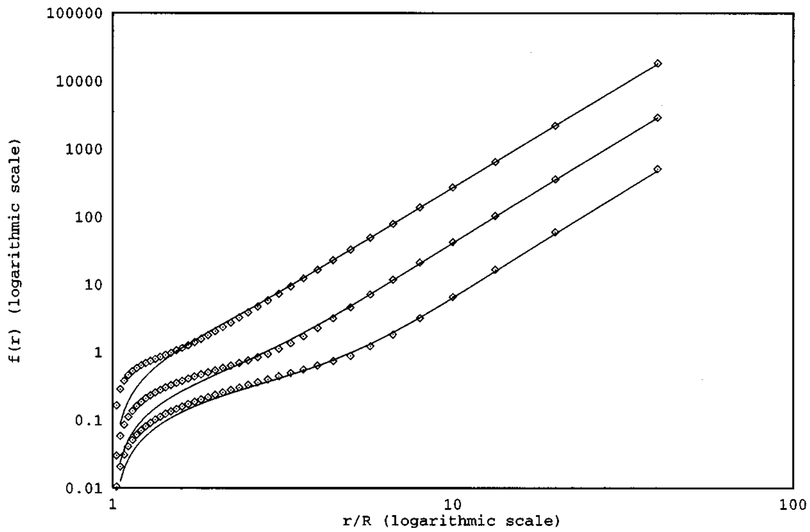

FIG. 5. Set of graphs for average $f(r)$ for several values of disclination radius, top to bottom: (a) $a/R=1.53$, (b) $a/R=2.07$, and (c) $a/R=5.05$ (note the log-log scale of the plot). Points are the result of numerical solution, solid lines are fits by the $f(r)$ from Eq. (13) with the above values of a for each graph. The logarithmic scale overemphasizes the error of the fit, which, in fact, is better than 10^{-5}.

(A2), it is possible to scale the time step with it, effectively taking the value $c=1$. The convergence appears to be robust and the steady-state solution $\beta(r,\theta)$ is quite independent of the starting director distribution.

After the exact distribution $\beta(\mathbf{r})$ is obtained, the next step is to check the validity of assumption (11) about the universal (for all values of the polar angle θ) function of the distance $f(r)$. Inverting that equation, we obtain the definition

$$f(r)=\frac{\tan 2(\theta-\beta)}{\sin 2\theta-\cos 2\theta \tan 2(\theta-\beta)}. \qquad \text{(A3)}$$

We then plot the variation of $f(r)$ for the whole range of angles θ, varying from 0 to $\pi/2$ (Fig. 4). Clearly, there is some nonuniversality in $f(r)$, but it is not very big and the error in using, as an approximation, the average function (the solid line in Fig. 4) appears to be insignificant.

Figure 5 presents a set of these ''approximately universal'' average functions $f(r)$ for several values of the disclination ring radius a/R. The plots are presented in the double-logarithmic scale and the far-field behavior $f(r)\to(r/a)^3$ is apparent. Note that, as it should be, the functions $f(r)$ pass through the points $f=1$ on the disclination ring $r=a$, and $f=0$ on the particle surface $r/R=1$. When the asymptotic far-field dependence of $f(r)$ is taken out, the difficult part is to approximate the sharp variation in the vicinity of the particle, in the region $R<r<a$. We use for fitting the general structure for $f(r)$, given by Eq. (13), where the coefficients $\mathcal{A}$,$\mathcal{B}$, and $\mathcal{C}$ are constrained by the ''boundary conditions'' (12). This determines two of these coefficients, for instance, $\mathcal{A}$ and $\mathcal{B}$ in terms of the one remaining ($\mathcal{C}$), which then is determined as a function of the parameter a/R to provide the best fit to all curves. There is certainly a lot of freedom on this last step; defining $\mathcal{C}(a)$ and some other expressions can be used with the same (or even better) degree of fitting accuracy. However, this choice appears to be irrelevant for the final energy calculation (Fig. 3) and we elected to use one of the simplest adequate expressions for $\mathcal{C}(a)$ [see Eq. (13)].

What is important in the result of this interpolation, is that the deviation of $f(r)$ from its far-field cubic asymptotic decays exponentially and is practically irrelevant outside the disclination ring. This makes the trial function $\beta(\mathbf{r})$, given by Eq. (10), a very good approximation of the director field around the radially anchored colloid particle suspended in a uniform nematic liquid crystal.

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